Multi-dimensional trading problem in multi-participant settings

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Abstract

The dimensionality of optimization problem arising within multi-market trading task grows exponentially with a growing number of markets. To prevent the dimensionality problem, multi-market trading is represented as a multi-participant decision making problem with finite common capital. Each local DM task is a single-market trading enriched by an ability to share a part of local capital with other local DM tasks (participants). The paper provides formulation of the problem and basic algorithmic steps. The approach is illustrated on the real market data.

1 Introduction

The trading task is a challenging problem for the most mathematics and economists. The trading is based on price speculation, where a trader (speculator) tries to buy cheep a contract, waits for price increase and then earns money by reselling the contract. The market speculation became a stochastic game, when speculator bets whether the price increase or decrease.

To design the trading strategy, speculators use different methods. These methods can be divided into two main categories: fundamental and technical analysis. The methods of fundamental analysis suppose that observed price is not reflecting the real price, but it is the only consequences of some real events in the world. Therefore the prediction is based on analysis of interest rates, inflation, the market state, actual news and activities of different institutions [1, 2, 3]. In contrast, methods of technical analysis, based 100 years ago [7], suppose that the observed price contains enough information to make a prediction, hence these methods primary deals with price sequences, so-called charting [4], indicators, such moving averages, and other measurable exchange variables.

Classical investing methods based on the fundamental or technical analysis serve primary for stock trading and are well developed for long-time investment in terms of decades, but we want to design the frequently trading system, therefore the methods are not suitable. Beside, the successful methods, if any, are not advertised everywhere and are kept in strict confidence. So up to the author's best knowledge, there is no known methodology how to design optimal strategy for speculators.

Our approach works with observed price sequences [5, 9] and can be classified as technical analysis. Recently several additional channels have been considered as well [8]. Our previous methods were designed for a single market with a restricting assumption of infinite capital to invest. That assumption allowed to solve a multiple markets trading problem market-by-market independently.

This paper deals with an extension of the task for constrained capital and multiple markets (Sec. 2). The dimensionality of obtained problem grows exponentially with a number of markets. To prevent the dimensionality problem, the problem is represented as multi-participant decision making problem with finite common capital. Each local DM task is a single-market trading enriched by an ability to share a part of local capital with other local DM tasks (Sec. 3). The approach is illustrated on the real market data (Sec. 4).

2 Trading task

The exchange of valuables primary serves to buying and selling of valuables. The exchange consists of *markets*. Each market is related to one type of valuables. We consider commodity contract exchange, therefore a *contract* represents the elementary tradeable part.

The market sets the price of contracts and allows to buy and sell them. A player on the exchange we are interested in is a *speculator*, i.e. a participant, who earns money not by owing of contracts but their reselling. The speculator can speculate for the price increase as well as its decrease. At the beginning of the trading cycle, the speculator declares, whether he speculates for the price increase or decrease, and declares the number of contract for this speculation. It is said to *open the position*. Then, the speculator waits for the price change (it is said to *stay in the position*). And finally, he declares, that he *closes the position*. The speculator earns money, when the price behavior follows his expectations, otherwise he looses. The gain or loss equals to the price change minus *transaction cost* for each contract held in the position. The speculator operates parallel at multiple markets.

2.1 Notation

We work with n markets, each denoted by $k \in \{1, 2, \ldots, n\}$. The time is discrete $t \in \{1, 2, \ldots, T\}$, where T is so-called trading horizon.

The variables denoted by the capital letters are vectors related to all n markets while variables related to the kth market are denoted by the lower-case letters and a subscript k. Both vectors and scalars related to time indexed by a subscript t.

Price $Y_t = (y_{1,t}, y_{2,t}, \ldots, y_{n,t})'$ is a vector of prices in time t.

Decision $U_t = (u_{1,t}, u_{2,t}, \ldots, u_{n,t})'$ is a vector of characterizing the positions chosen by the speculator. Each element $u_{k,t}$ characterizes the price direction and the number of contracts in a position. Position $u_{k,t}$ can be decrypted as follows: $u_{k,t} > 0$ for price increase, $u_{k,t} < 0$ for price decrease, and the absolute value $|u_{k,t}|$ gives the count of contracts in the position.

Transaction cost $P = (p_1, p_2, \ldots, p_n)'$ is a vector of a normalized transaction cost per contract. The transaction cost is paid for any change of the position, i.e. whenever $u_{k,t}$ changes according to $u_{k,t-1}$, it is paid $p_k|u_{k,t-1} - u_{k,t}|$ as transaction cost.

Capital C_t is a scalar variable characterizing the amount of the speculator money, the hard cash summed with a value of contracts held in open positions. The capital related to one market is denoted by $c_{k,t}$ and $C_t = \sum_{k=1}^{n} c_{k,t}$.

Gain G_t is a scalar variable characterizing gain or loss obtained at time t. The gain related to one market is $g_{k,t}$ and $G_t = \sum_{k=1}^{n} g_{k,t}$.

2.2 Task definition

The speculator enters the game with initial capital C_0. The exchange offers n markets to speculate and the speculator designs a decision $U_t = (u_{1,t}, u_{2,t}, \ldots, u_{n,t})'$. The decision is designed at each time t up to the trading horizon T, i.e. $t \in \{1, 2, \ldots, T\}$.

The gain obtained at each time t is:

$$G_t = (Y_t - Y_{t-1})' U_{t-1} - P' |U_{t-1} - U_t|, \tag{1}$$

where $Y_t = (y_{1,t}, y_{2,t}, \ldots, y_{n,t})'$ is a vector of prices at time t, $P = (p_1, p_2, \ldots, p_n)'$ is a vector of transaction costs. The speculator's capital C_t changes via

$$C_t = C_{t-1} + G_t = C_0 + \sum_{i=1}^{t} G_i.$$

At each time t the speculator cannot invest more than available capital, i.e., $Y_t'|U_t| \leq C_t$. Maximizing the capital under this constraint leads to the recommendation to bet all available capital to the most promising market. This is very risky, therefore to minimize the risk, the speculator distributes the capital between several markets. This is controlled by the second constraint $y_{k,t}|u_{k,t}| \leq M$ for $k \in 1, \ldots, n$, where the speculator sets the maximal capital M usable at one market.

Hence, the solved task can be written as the maximization of an expected value of the capital over the horizon C_T:

$$\max_{U_t} \mathcal{E}\left[C_T\right] = \max_{U_t} \mathcal{E}\left[C_0 + \sum_{i=1}^{T}(Y_i - Y_{i-1})'U_{i-1} - P'|U_{i-1} - U_i|\right] \tag{2}$$

where $\mathcal{E}[.]$ is an expected value. The constraints for the task have the following shape:

$$Y_t'|U_t| \leq C_t \tag{3}$$

$$y_{k,t}|u_{k,t}| \leq M \qquad \text{for } k \in \{1, \ldots, n\} \tag{4}$$

The absolute value $|.|$ in the gain function and the constraint (3) makes the optimization task growing with n. As a consequence, we should work with 2^n gain functions and $(2^n + n)$ constraints to design one decision vector U_t. The typical speculator works at 10-50 markets, which makes the task non-computable in real time. Therefore, to support multiple market trading on-line an alternative task formulation should be searched.

3 Multi-participant solution

Let us represent the n-dimensional trading task as a collection of n local one-dimensional DM tasks, where each designs DM strategy at one market.

The main issue of the proposed solution is in sharing the common capital. Whereas the multi-dimensional task optimizes the capital between all markets, the one-dimensional does not. Thus, an extra feature must be added to one-dimensional task to supply the capital sharing. To do this, one-market trading is considered as a virtual participant working at one market. The participant enriched by an ability to communicate with other participants and to offer his "redundant" capital or to ask for additional capital.

Note the bidding and asking of the capital should only supply the capital sharing, and the results are not expected to reach such good values as optimization results. But we expect the comparable quality without exponential growing of the overall problem.

3.1 Participants settings

This paragraph describes an algorithm of the solution.

3.1.1 One-market trading task

The initial capital C_0 is divided onto n parts $c_{k,0} = C_0/n$, $k \in \{1, \ldots, n\}$, which is given as initial capital to each participant. Then, the kth participant solves one-dimensional analogy of the original task (2)-(4), where the aim is to maximize the expected gain:

$$\max_{u_{k,t}} \mathcal{E}\left[\sum_{i=1}^{T}(y_{k,i} - y_{k,i-1})u_{k,i-1} - p_k|u_{k,i-1} - u_{k,i}|\right] \tag{5}$$

under the condition

$$y_{i,t}|u_{i,t}| \leq M. \tag{6}$$

The task (5) cannot be formulated by maximization of the capital $c_{k,t}$ (in analogy with (2)) because the participants can maximize their local capital by asking only, without any market transaction.

3.1.2 Capital sharing

The state of the k-participant's capital at time t can be expressed via:

$$c_{k,t} = c_{k,0} + \left(\sum_{i=1}^{t} (y_{k,i} - y_{k,i-1}) u_{k,i-1} - p_k |u_{k,i-1} - u_{k,i}| \right) - \left(\sum_{i=1}^{t} l_{k,t} \right) - p_k |u_{k,t}|. \quad (7)$$

The first term is initial capital. The second one is a gain obtained by contract reselling, it is one-dimensional analogy of the gain (1) summed over the time. The third one is a sum of *lent capital* $l_{k,t}$, which is the capital provided to other participants. The lent capital is one variable containing both capital borrowed from other participants ($l_{k,t} < 0$) and lent to other participants ($l_{k,t} > 0$). The fourth term is capital required to close the opened position. To close the position it is necessary to hold zero contracts and the fourth term expresses the transaction costs for the closing the position.

Given the present value of the capital $c_{k,t}$, the participant can perform two actions: to ask for $(c_{k,0} - c_{k,t})$, when $c_{k,t} < c_{k,0}$, or to bid $\alpha(c_{k,t} - c_{k,0})$, when $c_{k,t} \geq c_{k,0}$. α characterizes the ratio of bidding free capital, the setting of the coefficient allows to enhance or to reduce the capital flow between the participants. The ask cannot be modified by such a coefficient and its full value is used.

The pairing of bids and asks opens a wide area for research, we use one of the simplest approach. The biggest bidder sorts the askers from lowest ask to highest one. Then the bidder try to satisfy the lowest askers. Consequently, the second biggest bidder try continue in satisfying. Until there is some bid or ask. This approach tries to minimize the risk, when the capital is not enough. Moving the capital to the smallest asker can start his work and bring new money to the shared capital, whereas moving the capital to the biggest asker, the capital can be lost quickly. We expect that the quality of participants is characterized by their profit, therefore it seems better to help the participants with small loss.

4 Experiments

4.1 Settings

The experiments were performed for three markets ($n = 3$): Cocoa (CSCE), Petroleum-Crude Oil Light (NMX), and 5-Year U.S. Treasury Note (CBT). The available price data are sampled with a day period, at the end of trading day (so-called close prices). The used data were from January 1990 until October 2004, which makes 3700 samples.

The approximative solution of (2) and (5) is presented in [5, 6]. The solution extended by constraints (3) or (4) served as basic settings for the experiments.

To be most close to the original task, the lent coefficient was set to the maximal value, $\alpha = 1$, which should maximize the bid between the participants.

The maximal capital, which can be invested at one market, the constraints (4) and (6), were set to the value of \$50000 USD. Various values of initial capital C_0 were used, because the choice of C_0 influences the capital biding and asking. The initial capitals were selected to cover the both situation: each participant has less capital than may invest in one step ($C_0 < nM$), and each participant has more capital than he may invest ($C_0 > nM$). In the experiments with multidimensional optimization task (Sec. 2), the initial capital sets, which constraint is active more (constraint (3) or (4)).

4.2 Results

The quality of the obtained results was evaluated by the final gain, which is a difference between the final capital C_T and the initial capital C_0:

$$C_T - C_0 = \sum_{i=1}^{T} G_i.$$

Due to different initial capital C_0, the percent profit better characterizes the quality of obtained results:

$$PP = \frac{C_T}{C_0} \times 100\% - 100\%.$$

The results overview is in Table 1.

The original formulation (2)-(4) yields negative results for lower values of initial capital. This is caused by the active constraint (3), which importance decreases with the growing initial capital, while importance of the constraint (4) grows. The second reason for worse values is a fact, that the multi-dimensional optimization takes into account predictions for all markets and distributes the capital according to the ratio of predictions. This ratio changes often, which causes the higher transaction costs. With the growing initial capital, redistribution of the capital is not required, because capital is enough to have all position at maximal values allowed by the constraint (4) and the importance of the ratio between predictions decreases.

The multi-participants solution (Sec. 3) yields similar behavior at three lowest initial capitals, where the constraint (6) is not active. The better results with growing initial capital are caused by impossibility to invest the whole capital. The redundant capital is used to recover after un-successful operations. This principle is the same for the original task and the multi-participant formulation.

The comparison of the methods shows, that the multi-participants solution gives better results for lower values of initial capital and then slowly grows with the capital. The original multi-dimensional solution gives bad results for lower capital and then jumps to very good profit values and then the success decreases. Overall, we can say that the multi-participants solution performance is comparable with the optimal one.

The important aspect is also computing time. The complexity of the original task grows exponentially and for dimension $n = 3$ one experiment with 3700 samples is calculated in terms of hours, the same experiment with $n = 5$ is calculated about 1,5 day. To experimental testing there were 35 markets ($n = 35$) available. Therefore it is impossible to perform the optimization on all markets together. Whereas, the complexity of the multi-participants solution grows linearly with the n. The solution for $n = 3$ is computed in terms of seconds, for all 35 markets in terms of minutes. This property makes the multi-participant solution more suitable despite its a bit worse performance.

5 Conclusion

The multi-market trading task was presented with the comment of an exponential grow of its computational complexity. To avoid that, the multi-market trading is formulated as multi-participant decision-making problem with participants sharing the common finite capital by the biding and asking. The proposed algorithm was described and the obtained results were compared with the results of original multi-dimensional task. The comparison confirms the practical potential of the proposed solution.

The multi-participant solution can be improved by redefining multi-participants and their communication. We have more algorithms than the used [6]. The difference is in used type of model [8], prediction or optimization [9]. It can be interesting to compare the success participants working at one market, where each participant uses another algorithm. Moreover, the sharing of the capital between participants can be modified by adding the strategy of participant, which takes into account the previous success of another participant before lending the redundant capital. This extension can cause a deactivation of steady ineffective participants, whose only consume the money of successful ones.

	Gain		Percent profit	
C_0	Original	Participants	Original	Participants
10000	-5126	1153	-51,26%	+11,53%
35000	-14914	4036	-42,61%	+11,53%
60000	35295	6919	+58,83%	+11,53%
85000	48576	11854	+57,15%	+13,95%
110000	54738	19616	+49,76%	+17,83%
135000	60479	31879	+44,80%	+23,61%
160000	51748	38317	+32,34%	+23,95%

Table 1: Comparing of methods: the gain obtained in USD.

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References

[1] P. L. Bernstein. *The Curious History of Stock Prices and Interest Rates*. 1979.

[2] J. B. Cohen and E. Zinbarg. *Investment Analysis and Portfolio Management*. 1982.

[3] J. Ely and F. J. Robinson. *Finance*. New York, 1991.

[4] Stockcharts (http://www.stockcharts.org/).

[5] M. Kárný, J. Šindelář, Š. Pírko, and J. Zeman. Adaptively optimized trading with futures. Technical report, 2010.

[6] H. Matějková. Prediction of time series via functional expansion (bachleor thesis, in czech), 2010.

[7] Rhea R. *Dow Theory*. New York, 1932.

[8] J. Šindelář. Study of bvar(p) process applied to u.s. commodity market data. In Cemal Ardil, editor, *World Academy of Science, Engineering and Technology*, volume 58, pages 424–435, 2009.

[9] J. Zeman. Futures trading: Design of a strategy. In Cemal Ardil, editor, *World Academy of Science, Engineering and Technology*, volume 58, pages 951–955, 2009.